Comparing Sales Strategies Using the Markov Chain Relationship Model

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ABSTRACT

In this paper, the author applied the concept of the Markov chain and divided sales procedures into several indexes and states; use the state index for connecting success in sales and customer relations into Pfeifer’s method, establish a mathematical model, and demonstrate its result. In order to increase profits and decrease the cost of sales for the company, we further classify customers and propose different sale strategies. Case study and analysis are provided to elaborate the approach and its contribution to sales and CRM (customer relationship management) strategy.

Keywords: CRM, Relationship model, Selling process, Markov Chain

1. INTRODUCTION

Customer relationship modeling is always a popular topic for research. Sheng Peng conducted a thorough analysis, from the aspect of psychology, on the “customer person” in the process of selling.

He¹ considered that the selling is sales to people-to key customer people (KP, who are responsible for or have a direct controlling relation with the purchasing), no matter to a plant or a family. In the case, a selling is involved several factors, besides the enterprise’s “selling points”, customer’s “purchasing point”, more important, a good salesperson should also supply a KP with some “selling points” in KP’s group, those selling points should be not only accepted by the KP from his heart, but also can be accepted by KP’s group members, even more, some of the key ideas may let the KP believe the purchasing can bring him more respect in his group, and the KP is willing to declare the purchasing suggestion in the public. In such cases, the selling will be success in very large probability. He also thought that a sales process should be able to measured, it should never be managed in a “black box”, managers can’t handle the sales process and know the result only when it is out from the other side of the box. Such things happen because there is no state index set in the sales process, Sheng set up three indexes to help solving the problem.

Also to solve the problem, Reichheld, F.F⁶ consider that only one index is needed: “You simply need to know what your customers tell their friends about you”. Morgan⁷ and Kristensen⁸ do not agree with his idea.

With regard to the customer relations mathematic model and the aspect of analysis, though many papers involving in the topic, most of them are methods based on the contacting and sales result information analyzing, only a few of them are modeling inside the process. Blattberg and Deighton⁹ proposed an LTV(Life Cycle Value) model to compute the cost of acquiring and maintaining customers. In order to help managers to optimize their sales, Bronnenberg¹⁰ made an attempt to model and analyze consumer relationships using the Markov chain (MC); Based on supposed several states, Pfeifer and Carraway¹¹ constructed several CRM models with MC, and computed and analyzed different sales strategies within given parameters with profit as the objective, but they do not given more detailed computing and analysis to their model, also omitted that the supposed states can help salespeople improve their work; Jain and Singh¹² conducted a thorough survey of the area, describing the current and future trends.

In this paper we will introduce Sheng’s indexes and apply the indexes into Pfeifer’s¹¹ Markov model, then adopt different sales strategies, and a little detailed computing, to test the model, and analyze the benefit to the enterprise. In the analysis the customer classifying method will be employed, the results computed, and various scenarios compared.

2. THE SALES STATE INDEX, SELLING PROCESS, KNOWLEDGE, AND MARKOV MODEL

2.1 The Sales State Index, Selling Process Classification and Knowledge Management

In order to measure a sales process, Sheng¹¹ set up three index sets, based on the relationship between salesperson and KP, from connection relation, attitude to product, and confidence to product sides. Each index and states are given as following:

Connection index (To customer): Has nothing to say (low), and only says the official words (middle), the conversation is valid (good), and no secrets are kept from each other (high);

Attitude index (To product): basically not approve the product (low), basically approve the product (middle), and approve the product very much (high);

Confidence index (To product): approval given on a
Based on the concept given above, we can discuss

2.2 Markov Chain Modeling

Following large probability or selling in any chance?

process in to several states, each with its sales success

state and have larger sale s success probab ility; If your

result, what you should to do is to improve your relation

your state is three "middle", it is hard to say your sales

the selling will be success in very large probability; If

operating concept about states normalize salespeople’s

example, if your state is three "high" in the three sides,

but also supply them with an improved direction. For

just a customer, but a separa ted decision process. Its

monitor his people selling easily; locate the state and

standing to each given sale state; make sales managers

sales state description and unify salespeople’s under-

indexes classify a whole sales process into states, and its

customer purchasing point (PP), sales point in his group

such concept aims the data on KP’s information,

information. We think that a system built with such

why customer buy or not buy their product. So such

what they get is the reason, not a statistical estimation,

not just sales results, because its unified state description

the relationship between the salesperson and the customer

between the sales and its customers (we set states

-er), 4(no approval in one-on-one situations), 3(approval

situations), 1 (approval in big or important situations).

Obviously, state 1 holds the greatest probability of

success. These relation states within a sales process are

Sheng[1]).

Fig 1 A sale procedure with given sale states

process. The confidence index set divides a sales

Pfeifer’s model[4] is a model of the relationship

between the sales and its customers (we set states

the thought). When the trading began, the

relationship between the salesperson and the customer

can be in any state in Fig.1, and the sale can be

successful in any state, but the probability is different.

Suppose the income of a successful sale is N=40, and

and the cost of the sale is 4. The probability of one sale

succeed is the corresponding pi. If the sale succeeds,

the enterprise will obtain the benefit of N-M; if not, the

enterprise will lose M, and the probability of the

relationship going back one state is 1-pi. Obviously, pi≤
pj=1, i=1,2,…,j, with j as the state number, state 5

indicating no connection. Suppose also that the cost of

the sale is entered just before the deal is made.

The matrix P is a one-step transition matrix, R is a

reward vector, V is the expected net present value[4].

Here M1=M2=M3=M4=M. In fact, the element of

probability in the transition matrix (including in the result

matrix P2, P3, P4, etc.) represents the correspond-

ing relationship between enterprise and customer after

one or more sales. If required, we can set some

threshold value or make hypothesis test to those

elements which alert their relationship in case a

problem occurs or the relationship is broken.

To be more precise, suppose that there is a discount

to the present value after each purchase. Here, the
discount rate is d=0.2. Thus, the expected net present

value vector in j time of the sale, is(1):

\[ V' = \sum_{i=0}^{\infty} (1 + d)^{i} P^{i} R = (1 - (1 + d)^{-1}) P \cdot R \]

In order to simplify the results, we often use an infinite

horizon to describe the purchase. From (1), we can get

the expected net present value as follows:

\[ V = \lim_{j \to \infty} [I - (1 + d)^{-1} P]^{-1} R \]

Supposed i=4, p1=0.3, p2=0.2, p3=0.15, p4=0.05, then:

\[ V' = \left[ \begin{array}{c} 50.115 \\ 4.22 \\ 0.592 \\ -1.98 \\ 0 \end{array} \right] \]

\[ V = \left[ \begin{array}{c} 52.32 \\ 5.554 \\ 1.251 \\ -1.82 \\ 0 \end{array} \right] \]

The result shows that, in this case, 25% customer (in

state 1) makes the 90% profit of the enterprise. The

negative expected value –1.98, in state 4, means sales

enterprise shouldn’t sell in such customer relationship,
because the success probability is too small. How to solve the problem? Classifying customer is one way to improve the condition.

3. CLASSIFYING ANALYSIS

The purpose of the analysis is to determine the profit of the sale enterprise, and the object of the research is to see the effect produced by different sale strategies adopted to different customers, after having divided customers into different types. In many cases, the sale enterprise does not hope to gain much from the first or second trade, but hopes to make money over a given period of trading. Therefore, for ease of analysis, we assume an infinite case for the sale. In this case, the enterprise does not hope to gain much from the first or second trade, but hopes to make money over a given period of trading. Therefore, for ease of analysis, we assume an infinite case for the sale. In this case, the negative value in the V vector of equation 2 means that its income. We also transform equation 2 into 3 in the following analysis, to avoid the computation of d and the inverse operation in equation (2). We then have:

\[ [1-(1+d)^{-1}] P^T V = R \]  \hspace{1cm} (3)

Though there are 5 states in Fig.1, from the perspective of sales, there are only two kinds of customers: relationship and no relationship customers (or former customer\(^4\)).

3.1 Sales Strategies for Two Kinds of Customers

3.1.1 The General Result for the Two Kinds of Customers Scenario

From equation (3), the sale enterprise’s expected net present value equation is:

\[
\begin{bmatrix}
(1-p_1/p_4)v1 - (1-p_1/p_4)v2 \\
(p_2/p_4)v1 + v2 - (p_2/p_4)v3 \\
(p_3/p_4)v1 + v3 - (p_3/p_4)v4 \\
(1-p_4/p_4)v1 + v4 \\
0
\end{bmatrix} = \begin{bmatrix}
N-M \\
-M \\
-M \\
-M \\
0
\end{bmatrix}
\]

From equation (4) and (2'), we notice that the response of customers in different states to same input M is very different, \(p_1=0.3\), and \(p_4=0.05\). From Markov chain’s concept, we can see custom -ers in state4 as one time purchasing customers (OTPC), and ones in state1 as familiar ones. In order to compare easily, supposing we can increase sales cost to promote OTPC’s success rate, to think it as linear relation simply, \(M = (p1/p4)M\), the result shows, in one side, the cost to acquire a new customer is as much as \((p1/p4)\) times to retain a familiar one.

3.1.2 The Modified Strategy 1 for the Two Kinds of Customers Scenario

Under the given suppose, the better the relation is, the more the selling gets. The best relation customer will supply most of the sales profit. That means, under the given suppose, the better the relation is, the more the selling gets. The best relation customer will supply most of the sales profit.

\[ \begin{align*}
V1 &> V2 > V3 > V4 \\
\forall i &\in \{1,2,3,4\}, \sum_{i=1}^{N} v_i = \text{max}, \text{at least, each } v_i \geq 0 (i=1,2,3,4). \text{ From (5), each } v_i &\text{ is related to its } p_i, \text{ and in this case, } p_4 \text{ is the least, so if } v_4 \geq 0, \text{ then other } v_i \geq 0 (i=1,2,3). \text{ From (6), we can get the threshold } p_4 \text{ and } M \text{ as following:}
\end{align*} \]

\[ p_4 \geq \frac{M(1-p_1-q_1 p_2-q_1 q_2 p_3)}{N-M(1+q_1+q_2)} \]

The Fig2 shows the relation between them, to the given data, the threshold \(p_4\) in (2') should be \(p_4=0.09\), so in (2'), \(v_4<0\). By the same way, we can get each threshold \(p_i\) and \(M\) (\(i=1,2,3\)).

3.1.3 The Modified Strategy 1 for the Two Kinds of Customers Scenario

Under the given condition, Sales enterprise should modify its sales strategy to avoid losing money. The

\[ pi = \frac{p_i}{1-q_i}, qi = \frac{1-p_i}{1-q_i}, pi + qi = \frac{1}{1-q_i} (i=1,2,3,4). \]

\[ \begin{align*}
\text{From equation (4) and (2'), we notice that the response of customers in different states to same input } M &\text{ is very different, } p_1=0.3, \text{ and } p_4=0.05. \text{ From Markov chain’s concept, we can see custom -ers in state4 as one time purchasing customers (OTPC), and ones in state1 as familiar ones. In order to compare easily, supposing we can increase sales cost to promote OTPC’s success rate, to think it as linear relation simply, } M = (p1/p4)M, \text{ the result shows, in one side, the cost to acquire a new customer is as much as } (p1/p4) \text{ times to retain a familiar one.}
\end{align*} \]

\[ \begin{align*}
\text{If the aim is to obtain greater benefits, } \sum v_i &\text{ max , at least, each } v_i \geq 0 (i=1,2,3,4). \text{ From (5), each } v_i &\text{ is related to its } p_i, \text{ and in this case, } p_4 \text{ is the least, so if } v_4 \geq 0, \text{ then other } v_i \geq 0 (i=1,2,3). \text{ From (6), we can get the threshold } p_4 \text{ and } M \text{ as following:}
\end{align*} \]

\[ p_4 \geq \frac{M(1-p_1-q_1 p_2-q_1 q_2 p_3)}{N-M(1+q_1+q_2)} \]

\[ M \leq \frac{N^*p_4}{(1-q_1+q_1 q_2)\sum_{i=1}^{N} p_i+1-(1-p_1-q_1 p_2-q_1 q_2 p_3)} \]

The Fig2 shows the relation between them, to the given data, the threshold \(p_4\) in (2') should be \(p_4=0.09\), so in (2'), \(v_4<0\). By the same way, we can get each threshold \(p_i\) and \(M\) (\(i=1,2,3\)).

\[ \begin{align*}
\text{Under the given condition, } \sum v_i &\text{ max, at least, each } v_i \geq 0 (i=1,2,3,4). \text{ From (5), each } v_i &\text{ is related to its } p_i, \text{ and in this case, } p_4 \text{ is the least, so if } v_4 \geq 0, \text{ then other } v_i \geq 0 (i=1,2,3). \text{ From (6), we can get the threshold } p_4 \text{ and } M \text{ as following:}
\end{align*} \]

\[ p_4 \geq \frac{M(1-p_1-q_1 p_2-q_1 q_2 p_3)}{N-M(1+q_1+q_2)} \]

\[ M \leq \frac{N^*p_4}{(1-q_1+q_1 q_2)\sum_{i=1}^{N} p_i+1-(1-p_1-q_1 p_2-q_1 q_2 p_3)} \]

The Fig2 shows the relation between them, to the given data, the threshold \(p_4\) in (2') should be \(p_4=0.09\), so in (2'), \(v_4<0\). By the same way, we can get each threshold \(p_i\) and \(M\) (\(i=1,2,3\)).

3.1.3 The Modified Strategy 1 for the Two Kinds of Customers Scenario

Under the given condition, Sales enterprise should modify its sales strategy to avoid losing money. The
ers in state 4, and treat them as no relation customers.

Fig. 3 The state figure of Scenario (7)

When \( p_4 < M/v_1 \), in formula (5) \( v_4 < 0 \); i.e. sales to customers in state 4 (no approval in one-on-one cases) can never yield a profit. So stopping sales to them and treating them as customers in state 5 is a new sales strategy (see Fig.3, \( p_4 = 0 \), the corresponding \( M = 0 \), \( V_4 = 0 \)). Putting those results to formula (5), we have:

\[
V = \begin{bmatrix} v_1 & p_2 v_1 - M & 0 & 0 & 0 \\ v_2 & 0 & 0 & 0 & 0 \\ v_3 & 0 & 0 & 0 & 0 \\ v_4 & 0 & 0 & 0 & 0 \\ v_5 & 0 & 0 & 0 & 0 \\ \end{bmatrix}^T (7)
\]

Compared with formula (5), there is no negative \( v_4 \) in v3, which is in (7), so v3 in scenario (7) is better than that in (5); for the same reason, the v2 in scenario (7) is also better than that in (5). This means that, whether single vi or total profit(\( \sum i \)) is involved, scenario (7) is better than that in (5). If \( v_4 > 0 \), the conclusion is different. Considered the (2') data, we have the result:

\[
V = \begin{bmatrix} 53.149 & 6.621 & 2.644 & 0 & 0 \\ \end{bmatrix}^T, \text{it's better than (2')}.
\]

The result means that enterprise should not sell in the very small success probability case.

3.1.3 Improved Strategy 2 for Two Kinds of Customers

Furthermore, we can take no sales action for customers in state 3,4, and to treat them as no relation customers, as our strategy 2.

Fig. 4 The state in improved Scenario (9)

Similar to the previous analysis (Fig.3, also suppose \( p_3 = 0 \), corresponding \( M = 0 \)), taking the condition in formula (5), we get an improved scenario. In this case, the solution is:

\[
V = \begin{bmatrix} v_1 & p_2 v_1 - M & 0 & 0 & 0 \\ \end{bmatrix}^T (8)
\]

Compared with formula (7), the scenario is better than that of (7) only when \( v_3 > 0 \). Otherwise, it will be worse than scenario (7), because, in this case, \( v_3 \) can obtain profit. To the data given in (2'), we have result:

\[
V = \begin{bmatrix} 51.574 & 4.596 & 0 & 0 \end{bmatrix}^T, \text{the result is not as good as (7), for the \( p_3 \) in scenario (7) is great its threshold \( p_3 \).}
\]

From the above analysis, we can see that the sales strategy is available, because the sales enterprise can gain greater benefits, in the case of its environment not changed.

3.2 The Sales Strategy for Multiple Types of Customers

The case in Fig.3 is not realistic enough, because the enterprise should put some money into improving its relationships with its customer and bring it to a higher state: when the enterprise has made too few successful sales. Therefore, in order to reduce sales costs, we can divide both customers and sales costs into three types: sales cost M, retain cost \( H(<M) \), and 0.

3.2.1 Sales Strategy 1 for Three Kinds of Customers

A lower cost \( H \) is used for the case of improving customer relationships in order to reduce costs. Customers are classified into three types, such as 1,2,3,4; and 5; and the sales cost for state 4 is \( H \) (Fig.4).

The difference with that of earlier is that current \( p_4 \) is much great than before, because the current \( p_4 \) is not the probability of making a successful sale, but the state of increasing one. \( vi (i=1,2,3,4) \) is still the present net value of each state. From equation (3), we can obtain the sale enterprise’s expected net present value equation for the case of infinite purchases, as follows:

\[
\begin{aligned}
V_1 &= p_1(N - M) - (1 - p_3) M \\
V_2 &= p_1(N - M) - (1 - p_4) M \\
V_3 &= p_1(N - M) - (1 - p_3) M \\
V_4 &= p_1(v_1(N - M) - (1 - p_3) M)
\end{aligned}
\]

The transition matrix \( P \) and reward vector is:

\[
\begin{bmatrix}
p_1 & 0 & 0 & 0 & 1 - p_1 & 0 & 0 & 0 \\
0 & p_2 & 0 & 0 & 0 & 1 - p_2 & 0 & 0 \\
0 & 0 & p_3 & 0 & 0 & 0 & 1 - p_3 & 0 \\
0 & 0 & 0 & p_4 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

From equation (3), we obtain the expected net present value as follows. The symbols are the same as those in

\[
V = \begin{bmatrix} (N-M)qM & (N-M)qM & (N-M)qM & v_3 - H & 0 \end{bmatrix}^T (9)
\]

(6). Because formula (7) can be shown as following:

\[
V = \begin{bmatrix} v_1 & (p_2 q_2 p_3 v_1 - (1 + q_2) M) & p_3 v_1 - M \end{bmatrix}^T
\]

and to prove which is larger((7) or (9)) is a little hard,
scenario (9), the other two is: \( v_1 = 44.89, v_2 = 40 \), when \( p_3 \) varying from 0 to 0.4, so \( \sum v_i \) in scenario (9) is much better than that \( \sum v_i \) in scenario (7) (note: we haven't consider the v4 in scenario (9) in the analysis).

\[ V = [44.89, 40.00, 37.90, 49.47, 0] \] is the (9) result with the data in (2'), except \( p_4 = 0.3 / 1.2 \).

The result means that enterprise should improve the relationship with customer first, and reduce the sales chance under too low success probability case.

### 3.2.2 Characteristics of the Transition Matrix

In a transition matrix with structure as such \( P \), all customers will finally become no connection customers. Because when \( p_{ij} \geq 0 \) in the matrix \( P \), there must be a steady limit probability vector. Suppose this vector is \( W \) and that \( W = (W_1, W_2, W_3, W_4, W_5) \). We then have a steady limit probability equation \( W = P^T \cdot W \). For the character of matrix \( P \), we obtain the first two lines as:

\[
\begin{align*}
\begin{bmatrix}
1 & 2 & 3 & 4 & 5
\end{bmatrix} & = (1 - p_1) \begin{bmatrix} W_1 \end{bmatrix} & & & \\
(1 - p_2) W_2 & & (1 - p_3) W_3 & & \\
(1 - p_4) W_4 & & & (1 - p_5) W_5 & \\
& & & & (1 - p_6) W_6
\end{align*}
\]

Fig. 6 The state in an improved Scenario (10)

The result means that enterprise should improve the relationship with customer first, and reduce the sales chance under too low success probability case.

### 3.2.3 Sales Strategy 2 to Three Kinds of Customers

We can improve our sale strategy further, and propose a more conservative sales strategy: the enterprise should only conduct its sales in state 1,2.

The sales enterprise may require a regulation that the product must be sold at the high relation state, i.e. where there is a high probability that the sale will be a success—in “approval in small catcall situations” and “approval in big or important situations.” Therefore, by inputting \( H \) sales cost, each success in state 3,4 can increase the step of the state forward by one state; each failure can set the relationship back by one state; and the sales action can only occur in states 1 and 2 with a cost of \( M \). This strategy is exactly like the one that classifies customers into the following three types: 1,2; 3,4; 5, only now Fig.5 is changed to Fig.6. This is a typical random walk model. Hence, we can establish the equation by using formula (3):

\[
\begin{align*}
v_1 &= p_1(N - M) - (1 - p_1)M_1 \\
v_2 &= p_2(N - M_2) - (1 - p_2)p_3M_2 \\
v_3 &= p_3(v_2 - H) - (1 - p_3)H \\
v_4 &= p_4(v_3 - H) - (1 - p_4)H
\end{align*}
\]

The transition matrix \( P \) and reward vector are:

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[ R = \begin{bmatrix}
N - M_1 \\
N - M_2 \\
v_2 - H \\
v_3 - H \\
v_4 - H
\end{bmatrix}
\]

The expected net present value is equation (3) after the purchase (Suppose \( M_1 = M_2 = M \)).

\[ V = \left[ \frac{v_1}{p_1} \frac{v_2}{p_2} \frac{v_3}{p_3} \frac{v_4}{p_4} \right] = \begin{bmatrix}
44.89 \\
40.00 \\
37.90 \\
49.47 \\
0
\end{bmatrix}
\]

Comparing with scenario (9), there is no change in \( v_1, v_2 \) of scenario (10). The value of \( p_3, p_4 \) changes (the change will result in \( v_3 \)'s numerator becoming larger and its denominator becoming smaller), but the change in \( v_3 \) results in a change in \( v_4 \). If we can prove \( v_3 \) in (10) is better than \( v_3 \) in (9), then we can say scenario (10) is better than scenario (9). We have:

\[ \frac{v_2(N - (1 + q)M)}{1 - p_3} - \frac{v_3(N - (1 + q)M)}{1 - p_3} = \frac{v_2 - N}{1 - p_3} \]

Notice that, we use \( M \) instead of \( H \) for the comparing.

\[ v_2 - N = \frac{v_2(N - (1 + q)M)}{1 - p_3} - \frac{v_3(N - (1 + q)M)}{1 - p_3} \]

When \( \frac{p_2}{1 - q} = \frac{M}{N} \), the \( v_3 \) in (10) is better than \( v_3 \) in (9). Consider \( H \), we can say that the scenario (10) is better than scenario (9). Also we can get result:

\[ V = [44.89, 40.00, 51.64, 67.79, 0] \] (Here: \( p_4 = p_3 = 0.3 / 1.2 \)), it looks better than that in scenario (9).

### 4. CONCLUSION

In the paper, we analyzed the customer relationship set up the sales state index for the sales process, and modeled the relationship by introducing the concept of the MC based on the given state index; and presented the results of our analysis of the model. Pfeifer [4] pointed out that such an approach can also be applied to analyzing the problem of “Recency,” “Frequency,” and “Monetary.”

The proposed approach here is to model the relationship with the MC by using various indexes, analyze the relationship between cost and the probability of making a successful sale in terms of benefits obtained, test different sales strategies on the model and present the corresponding solution.

The analysis results show us that the most conservative sales strategy is much better than the normal strategy.
That means, in the case of small sales success probability, the sales enterprise shouldn’t sell in any state at any chance, but to improve the relationship with customer first and prepare to sell in the high success probability state.

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REFERENCES