Inventory Management with the Internet-Based Direct Channel in a Two-Echelon Supply Chain System

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Abstract

We present a two-echelon dual-channel inventory model in which stocks are kept in both the manufacturer warehouse (upper echelon) and the retail store (lower echelon), and the product is available in two supply channels: the traditional retail store and the web-based direct channel. The system receives stochastic demand from two customer segments: those who prefer the traditional retail store and those who prefer the web-based direct channel. Any order placed through the direct channel is fulfilled through direct delivery from the manufacturer warehouse. When a stockout occurs in either channel, customers are willing to shift the channel with a known probability. Customers who are unwilling to shift the channel result in lost sales. In order to develop operational measures of supply chain flexibility, we define a cost structure which captures two different operational cost factors: inventory holding costs and lost sales costs. Several insights are evident from the numerical experiments. We also examine the performance of two other possible channel strategies: retail-only and direct-only strategies. Simulation outcomes indicate that the dual-channel strategy outperforms the other two channel strategies in most cases, and the cost reductions realized by the flexibility of the dual-channel system could be very significant.

1. Introduction

The advent of the Internet has prompted many manufacturers to redesign their traditional channel structures by engaging in direct sales. Consider a two-echelon inventory system that consists of a manufacturer with a single warehouse at the top echelon and a retail store at the bottom echelon. Suppose that orders placed online are filled through direct delivery from the top echelon. Opening a web-based direct channel alongside the existing retail store may cause havoc on the product demand structures, and thus requires the company to redesign the optimal inventory allocations. In this paper, we construct an analytical model and define a cost structure which captures the inventory-related operational costs to evaluate the performance of a two-echelon dual-channel supply system.

The pervasive presence of multi-echelon inventory systems throughout the business world has been recognized in a long time. The concept of echelon stock was first introduced by Clark and Scarf (1960). Inventory control in multi-echelon systems is known to be a challenging research area. Because of the complexity and intractability of the multi-echelon problem, Hadley and Whitin (1963) recommend the adoption of single location, single echelon models for the inventory systems. Sherbrooke (1968) constructs the METRIC model, which is capable of identifying the optimal stock levels that minimize the expected backorders at the locals subject to a budget constraint. This model has been considered as the first multi-echelon inventory model for service parts. Thereafter, a large set of models that generally seek to identify optimal lot sizes and safety stocks in a multi-echelon framework were produced by many researchers (e.g., Deuermeyer and Schwarz 1981, Moizadeh and Lee 1986, Svoronos and Zipkin 1988, Axsäter 1993, Nahmias and Smith 1994, Aggarwal and Moizadeh 1994). In addition to analytical models, simulation models have also been developed to capture the complex interactions of the multi-echelon inventory systems problem (e.g., Clark and Trempe 1983, Pyke 1990, Dada 1992, Alfredsson and Verrijdt 1999).

The study of multi-channel supply chains in the direct versus retail environment emerged only recently. The focus of this stream of literature is on the channel competition and coordination issues in the setting where the upstream echelon is at once a supplier to and a competitor of the downstream echelon (e.g., Rhee and Park 1999, Tsay and Agrawal 2001, Chiang, Chhajed and Hess 2002). These papers present the dual-channel design problem by modeling the price and/or service interactions between upstream and downstream echelons.

The theoretical basis for multi-echelon dual-channel inventory problem has not yet been well developed. Toward this propose, we incorporate the direct channel into a traditional two-echelon inventory system to determine the optimal inventory levels for each echelon.

2. The Two-Echelon Dual-Channel Inventory Model

Consider a two-echelon dual-channel supply system that consists of a manufacturer with a single warehouse at the top echelon and a retail store at the bottom echelon. Figure 1 illustrates the topology and product flows of the system. The complete set of assumptions made for the inventory model is listed below.
2.1 Model Assumptions

(1) The product is available for customers in both the retail store and the direct channel. Any order placed through the direct channel is fulfilled through direct delivery from the manufacturer warehouse. Demands across the two different channels are independent, and the total demand rate for the product in the system is \( \lambda \). Specifically, customers are assumed to arrive at the retail store according to a Poisson process with constant intensity \( \lambda_{s} = (1-\alpha)\lambda \), and orders placed through the direct channel are in accordance with a Poisson process with rate \( \lambda_{d} = \alpha \lambda \), where \( \alpha \), the direct channel preference rate, is defined as the proportion of potential consumers who would prefer buying directly from the manufacturer.

(2) When the retail store is out of stock, customers are willing to shift from the retail store to the direct channel with a known probability called the retail channel diversion rate, \( \beta_{s} \). Similarly, when the manufacturer warehouse is out of stock, customers are willing to shift from the direct channel to the retail store with a known probability called the direct channel diversion rate, \( \beta_{d} \). When a stockout occurs in either channel, customers who are unwilling to shift the channel result in lost sales. Customers are assumed to be lost when both the retail store and the manufacturer warehouse are out of stock.

(3) The warehouse replenishment lead-time and the retail store replenishment lead-time are assumed to be exponentially distributed with mean \( 1/\mu_{w} \) and \( 1/\mu_{r} \), respectively. A one-for-one replenishment inventory policy is applied, and replenishment backorders from the retail store are allowed at the manufacturer warehouse. A customer served from stock on hand will trigger a replenishment order immediately by EDI (electronic data interchange). Therefore, the information lead time is assumed to be zero. Under this replenishment inventory policy, the inventory position is kept constant at a base-stock level. The base-stock levels at the warehouse and the retail store are denoted by \( S_{w} \) and \( S_{r} \), respectively.

2.1 The Markov Model

Based on the assumptions in the previous section, the corresponding Markov model can be constructed with the state space \((x, y)\), where

\[
x = \text{stock on hand at the manufacturer warehouse}, \quad -S_{w} \leq x \leq S_{m}, \quad \text{and} \quad y = \text{stock on hand at the retail store}, \quad 0 \leq y \leq S_{r}
\]

Note that the stock on hand at the manufacturer warehouse can be negative due to the replenishment orders from the retail store. There are four events that lead to a change of state: 1) a customer arrives at the retail store, 2) an order is placed through the direct channel, 3) a replenishment order arrives at the manufacturer warehouse, and 4) a replenishment order arrives at the retail store. Let \( \pi_{xy} \) denote the steady-state probability that \( x \) items are on hand at the manufacturer warehouse and \( y \) items are on hand at the retail store. Then we can find the steady-state probabilities by solving the linear equation system that contains the balance equations and the normalizing constraint.

3. Cost Structure

In this section, we conduct an economic analysis of the model to evaluate the performance of the two-echelon dual channel system. We define a cost structure that takes into account two different operational cost factors, inventory holding cost and lost sales cost. In the sections that follow, these cost factors are specified in terms of the steady-state probabilities.

3.1 Inventory Holding Cost

Let \( h_{w} \) and \( h_{r} \) be the inventory holding costs in dollars incurred by the firm per item per time unit at the manufacturer warehouse and the retail store, respectively. Then, given the steady-state probabilities, the total inventory holding cost, \( C_{H} \), is determined by

\[
C_{H} = h_{w} \sum_{j=1}^{S_{w}} \sum_{i=0}^{S_{r}} x \pi_{xy} + h_{r} \sum_{j=S_{w}+1}^{S_{r}} \sum_{i=1}^{S_{r}} y \pi_{xy}.
\]

Clearly, the first portion of \( C_{H} \) captures the holding cost from the manufacturer warehouse, while the second part reveals the holding cost from the retail store.

3.2 Lost Sales Cost

Recall that when a stockout occurs in either channel, customers who are unwilling to shift the channel result in lost sales. Moreover, customers are lost when both the retail store and the manufacturer warehouse are out of stock simultaneously. Assume that the cost of losing a customer is \( l \) per customer. Then the total lost sales cost incurred by the firm, denoted by \( C_{L} \), can be determined by the steady-state probabilities, and is given by
using one single channel), we learn that when obtained by using dual-channel strategy (compared to reduction. If we calculate the cost reductions that are customers is close to the number of retail customers, the value of \( \alpha \) is around 0.5, that is, when the number of direct whichever results in a lower cost). We see that when the cost from using dual channels to the total cost from using other two channel strategies for all values of \( \alpha \).

In most cases the dual-channel strategy outperforms the retail-only strategy and direct-only strategy. In order to find the base-stock levels that minimize the total cost.

### 4. Comparison of Channel Performance

Instead of using both the retail store and the direct channel, a firm can just use either one of the two channels to fulfill the demand. After evaluating the performance of the dual-channel strategy, we also investigate the performance of the other two channel strategies, retail-only strategy and direct-only strategy. In order to compare the channel performances, the numerical results from the two demand fulfillment strategies are juxtaposed with the results from the dual-channel strategy discussed before. The results show that the total cost of using the retail-only strategy increases in the direct channel preference, \( \alpha \), while on the other hand, the total cost of using the direct-only strategy decreases in \( \alpha \). We find that in most cases the dual-channel strategy outperforms the other two channel strategies for all values of \( \alpha \).

Figure 2 illustrates the cost of using dual channels relative to using one single channel (the ratio of the total cost from using dual channels to the total cost from using either the retail store or the direct channel, depending on whichever results in a lower cost). We see that when the value of \( \alpha \) is around 0.5, that is, when the number of direct customers is close to the number of retail customers, the dual-channel strategy could lead to an impressive cost reduction. If we calculate the cost reductions that are obtained by using dual-channel strategy (compared to using one single channel), we learn that when \( \beta_d = \beta_d = 0.75 \), the average cost reduction for all values of \( \alpha \) is 54%, and when \( \beta_r = \beta_d = 0.25 \), the average cost reduction for all values of \( \alpha \) is 73%. It is clear that when the values diversion rates are low, the cost reductions realized by the dual-channel strategy are remarkable. The result reflects the fact that, when customers are less willing to deviate from their desired channel, using dual channels helps to reduce the number of losing customers, and thus leads to a lower total cost.

### 5. Concluding Remarks

The two-echelon dual-channel inventory model presented in this paper is constructed based on queuing theories. We analytically develop operational measures of supply chain flexibility. We define a cost structure which captures two different operational cost factors: inventory holding cost and lost sales cost. Our analysis of the model leads to an exact evaluation of system performances.

Several variations of our performance evaluation procedure are possible. For example, we have used the inventory-related cost as a criterion to evaluate the channel performance. However, sometimes achieving a high customer service level is an important objective for the system. In this case, we can easily include fill rates in the constraints or in the objective of the system optimization model for the performance evaluation.

We are not aware of any inventory model that handles the multi-echelon system with multiple channels receiving demand from different market segments. Our model has provided some insights in this important line of inquiry. However, due to the complex nature of the problem, the model is subject to some assumptions and therefore, has some limitations. For example, the results from our model may not hold if different inventory policies are applied. A more extensive investigation in this area is warranted.

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### References


