Upstream R&D Competition and Cooperation in a Two-Tier Supply Chain

Pei-Jun Huo, Jian Chen
School of Economics and Management
Tsinghua University
Beijing 100084, China
{huopj, chenj}@em.tsinghua.edu.cn

Ji-Xiang Chen
Department of Business Administration
Shanghai Jiao Tong University
Shanghai 200052, China
jixichen@fm365.com

Abstract

This paper derives a two-tier supply chain model with many firms in each tier. The upstream firms engage in cost-reducing R&D activities. Under the case of R&D competition, the paper discusses how the changes in R&D spillover, R&D efficiency and the numbers of the manufacturers and the suppliers affect the R&D expenditures, the quantities and the profits. Then, R&D cooperation is considered. And what effects arise with the changes in the R&D spillover and the degree of cooperation are studied.

Key words: supply chain, upstream firm, downstream firm, R&D competition, R&D cooperation

1. Introduction

The subject of R&D competition and cooperation has gained a lot of attention from researchers in a variety of settings. Such literature usually uses a two-stage game that firms choose their R&D expenditures before making decisions on production. [1] examines the impact of multinational presence on domestic firms’ innovative efforts in a model focusing on the strategic dimension of R&D. [2] addresses the question about the optimal degree of spillovers and the number of rival firms necessary for obtaining the maximum amount of effective R&D. Both [1] and [2] only consider R&D competition between firms. Meanwhile, many papers consider R&D competition and cooperation simultaneously and make comparisons between them. [3] presents an interesting analysis of cooperative and non-cooperative R&D and compares the R&D investment and output under cooperative R&D with those under non-cooperative R&D. [4] extends the results in [3] to the case of more than two firms and more general demand and cost assumptions. [5] extends the analytical framework in [3] to a two-industry, two-firm-per-industry model allowing for R&D spillovers to occur within industries as well as between industries. [6] compares duopoly outcomes between two alternative models of independent R&D and non-cooperative RJVs, where there are complementarities between firm-specific R&D resources.

Besides the two-stage models, some papers introduce another stage game before the other games occur. [7] derives the non-cooperative, optimal policy towards international R&D cooperation. In the model, the governments simultaneously announce their R&D subsidy rates before the firms choose their R&D expenditures and quantities. [8] examines the impact of the firms’ mode of foreign expansion on the incentive to innovate as well as the effects of R&D activities and technological spillovers on the firms’ international strategy. [8] considers a two-country imperfect competition model where the firms face three different type of decisions: how to expand abroad, how much to spend on R&D and how much to sell in each market.

Although there exist a lot of papers about R&D competition and cooperation, we can hardly find one under the case of supply chain. An exception may be [9] which derives a model that an integrated firm produces the input and engages in R&D to reduce the input production cost. The integrated firm sells the input to its rival at a regulated price and competes with it in the final product market. [9] examines input price regulation’s effects on R&D and output. In our paper, we consider a two-tier supply chain with many firms in each tier. The upstream firms, the suppliers, conduct R&D activities that result in reduction in marginal cost. We use a parameter $\beta$ to capture R&D spillovers between the
suppliers. We first consider R&D competition and then R&D cooperation. Concerning R&D competition, we discuss how the changes in R&D spillover, R&D efficiency and the numbers of the manufacturers and the suppliers affect the R&D expenditures, the quantities and the profits. Under R&D cooperation, we study what effects arise with the changes in the R&D spillover and the degree of cooperation.

The two-tier-supply-chain structure with many firms in each tier in the paper is similar to that of [10]. Such kind of structure also appears in [11] and [12]. [10] examines the impact of fixed and variable costs on the structure and competitiveness. [11] examines vertical integration as an equilibrium phenomenon and consider the issue of private profitability versus collective profitability. [12] analyses the effects of different institutional arrangements of union-firm bargaining.

2. The Model

Consider a two-tier supply chain with \( n_1 \) firms, the manufacturers, in the downstream tier, and \( n_2 \) firms, the suppliers, in the upstream tier. The firms in the same tier engage in Cournot competition. The suppliers sell a homogeneous input at price \( p_2 \) to the manufacturers which use it to produce a final product. Without loss of generality, we assume that one unit of final product requires one unit of input. The inverse demand function of the final product is \( 11bQap \), where \( p \) is the price and \( Q \) is the total output. Denote the output of manufacturer \( i \) as \( q_{1,i} \), then \( Q = \sum_{i=1}^{n_1} q_{1,i} \). The profit of a representative manufacturer is

\[
\Pi_{1,i} = (p_1 - v_1 - p_2)q_{1,i} \tag{1}
\]

where \( v_1 \) is the constant marginal cost of the final product.

The suppliers conduct R&D activities that result in reduction in marginal cost. Let \( x_j \) be the level of R&D investment undertaken by supplier \( j \) and let \( v_{2,j} \) denote supplier \( j \)'s marginal cost. In order to model the possibility of imperfect appropriability (i.e., technological spillovers between the suppliers), we introduce a spillover parameter \( \beta \in [0,1] \). This means that the magnitude of supplier \( j \)'s cost reduction is determined by its own technological knowledge and by a fraction \( \beta \) of the sum of the other suppliers’ knowledge. More specifically

\[
v_{2,j} = v_{2,0} - \sqrt{g_{x,j}} - \beta \sum_{\substack{m=1 \backslash m \neq j}}^{n_2} \sqrt{g_{x,m}}
\]

(2)

where the parameters \( g \) and \( v_{2,0} \) describe the efficiency of the R&D process and the initial marginal production cost of the suppliers. The expression \( \sqrt{g_{x,j}} \) is an R&D production function which reflects the existence of diminishing return to R&D expenditures and can be seen in many papers\([3][8][13][14]\). The profit of supplier \( j \) is

\[
\Pi_{2,j} = (p_2 - v_{2,j})q_{2,j} - x_j \tag{3}
\]

where \( q_{2,j} \) is supplier \( j \)'s output.

Now, we consider the decision problem of the manufacturers. Manufacturer \( i \) chooses its output to maximize its profit by taking the other manufacturers’ output as given. The first-order condition can be derived from (1), which is

\[
a - v_1 - p_2 - b(Q_1 + q_{1,i}) = 0
\]

Since the manufacturers are identical, we can get

\[
q_{1,i} = q_1 = \frac{a - v_1 - p_2}{b(n_1 + 1)}
\]

and
\[ Q_i = n_i q_i = \frac{n_i(a - v_i - p_2)}{b(n_i + 1)} \]  

(4)

In equilibrium, the overall output of the manufacturers and the suppliers must be equal. Substituting \( Q_2 \) for \( Q_1 \), (4) can be rearranged to

\[ p_2 = (a - v_i) - \frac{b(n_i + 1)}{n_i} Q_2 \]  

(5)

where \( Q_2 = \sum_{m=1}^{n_2} q_{2,m} \) is the overall output of the suppliers. Supplier \( j \) chooses \( q_{z,j} \) to maximize its profit. From (3) and (5) we get the first-order condition as follows

\[ a - v_i - v_{z,j} - \frac{b(n_i + 1)}{n_i} (Q_2 + q_{z,j}) = 0 \]  

(6)

We can get \( n_2 \) equations from (6) since \( j \) varies from 1 to \( n_2 \). Then, we can, respectively, derive the suppliers’ total output and supplier \( j \)’s output as follows

\[ Q_2 = \frac{n_i}{b(n_i + 1)(n_2 + 1)} [n_2 (a - v_i) - \sum_{m=1}^{n_2} v_{z,m}] \]  

(7)

\( q_{2,j} = \frac{n_i}{b(n_i + 1)(n_2 + 1)} [(a - v_i) - (n_2 + 1) v_{2,j} + \sum_{m=1}^{n_2} v_{z,m}] \]  

(8)

Substituting (5), (7) and (8) into (3), we get

\[ \Pi_{2,j} = \frac{n_i}{b(n_i + 1)(n_2 + 1)} [(a - v_i) - (n_2 + 1) v_{2,j} + \sum_{m=1}^{n_2} v_{z,m}]^2 - x_j \]  

(9)

3. R&D Competition

In this section, we first derive the equilibrium when the suppliers independently decide R&D expenditures to maximize their individual profits. Then we discuss what effects arise with changes in some parameters.

Substituting (2) into (9) results in

\[ \Pi_{2,j} = \frac{n_i}{b(n_i + 1)(n_2 + 1)} [(a - v_i - v_j^0) + (n_2 + \beta - n_2 \beta) \sqrt{gx_j} + (2 \beta - 1) \sum_{m \neq j} \frac{g_{x,m}}{\sqrt{gx_j}}] - x_j \]  

(10)

Supplier \( j \) chooses its R&D investment, \( x_j \), to maximize its profit. The first-order condition is

\[ \frac{\partial \Pi_{2,j}}{\partial x_j} = \frac{n_i}{b(n_i + 1)(n_2 + 1)} [(a - v_i - v_j^0) + (n_2 + \beta - n_2 \beta) \sqrt{gx_j} + (2 \beta - 1) \sum_{m \neq j} \frac{g_{x,m}}{\sqrt{gx_j}}] - 1 = 0 \]

from which we get

\[ \sqrt{gx_j} = \sqrt{gx} = \frac{n_i g(n_2 + \beta - n_2 \beta) (a - v_i - v_j^0)}{b(n_i + 1)(n_2 + 1)^2 - n_i g(1 - \beta + n_2 \beta)(n_2 + \beta - n_2 \beta)} \]

or

\[ x_j = x = \frac{n_i^2 g(n_2 + \beta - n_2 \beta)^2 (a - v_i - v_j^0)^2}{[b(n_i + 1)(n_2 + 1)^2 - n_i g(1 - \beta + n_2 \beta)(n_2 + \beta - n_2 \beta)]^2} \]

(11)

We need the following condition

\[ b(n_i + 1)(n_2 + 1)^2 > n_i g(1 - \beta + n_2 \beta)(n_2 + \beta - n_2 \beta) \]

(12)

to ensure that the discussions are practical.

Now, we get

\[ \Pi_{2,j} = \Pi = \frac{n_i [b(n_i + 1)(n_2 + 1)^2 - n_i g(n_2 + \beta - n_2 \beta)^2 (a - v_i - v_j^0)^2]}{[b(n_i + 1)(n_2 + 1)^2 - n_i g(1 - \beta + n_2 \beta)(n_2 + \beta - n_2 \beta)]^2} \]

(13)

\[ Q_1 = Q_2 = Q = \frac{n_i n_2 (n_2 + 1)(a - v_i - v_j^0)}{[b(n_i + 1)(n_2 + 1)^2 - n_i g(1 - \beta + n_2 \beta)(n_2 + \beta - n_2 \beta)]} \]
\[ q_{i,j} = q_i = \frac{n_j(n_j + 1)(a - v_i - v_j)}{[b(n_i + 1)(n_j + 1) - n_j g(1 - \beta + n_j \beta)(n_i + \beta - n_j \beta)]} \]

\[ q_{z,j} = q_z = \frac{n_j(n_j + 1)(a - v_i - v_j)}{[b(n_i + 1)(n_j + 1) - n_j g(1 - \beta + n_j \beta)(n_i + \beta - n_j \beta)]} \]

\[ \Pi_{i,j} = \Pi_i = bq^i \]

**Proposition 1** Each supplier’s R&D expenditure increases with R&D efficiency and the number of the manufacturers and decreases with the number of the suppliers.

**Proof** From (11) we can immediately get \( \frac{\partial x}{\partial g} > 0 \) and \( \frac{\partial x}{\partial n_i} > 0 \). Thus, the R&D effort of the suppliers increases if R&D activities are more efficient or there are more manufacturers.

Partially differentiating (11) with respect to \( n_z \), we get

\[ \text{Sign}\left(\frac{\partial x}{\partial n_z}\right) = \text{Sign}\{n_i g(n_z + \beta - n_z \beta)^2 - b(n_i + 1)(n_z + 1)^2 n_z - n_z \beta + 3 \beta - 1\} \]

After some simple manipulations, we can show that

\[ \frac{b(n_i + 1)(n_z + 1)(n_z - n_z \beta + 3 \beta - 1)}{1 - \beta + n_z \beta} \geq \frac{b(n_i + 1)(n_z + 1)^2(n_z + \beta - n_z \beta)\beta}{1 - \beta + n_z \beta} \]

always holds since it is equivalent to \((n_z - 1)(2 \beta - 1)^2 \geq 0\), which is undoubtedly true.

Substituting the inequty in (12) into (15) yields

\[ b(n_i + 1)(n_z + 1)(n_z - n_z \beta + 3 \beta - 1) \geq b(n_i + 1)(n_z + 1)^2(n_z + \beta - n_z \beta)\beta \]

which leads to \( \frac{\partial x}{\partial n_z} < 0 \) from (14). Thus, each supplier invests less in R&D if more firms exist in the upstream tier.

**Proposition 2** (1) For large spillovers (\( \beta \geq 0.5 \)), the suppliers reduce their R&D expenditures when the spillover increases. (2) We can not unambiguously indicate how the R&D investments vary with the spillover if \( \beta < 0.5 \). However, the less the numbers of the manufacturers and the suppliers, the lower the R&D efficiency and the higher the spillover are, the more likely that the R&D investments decrease.

**Proof** Partially differentiating (11) with respect to \( \beta \), we have

\[ \text{Sign}\left(\frac{\partial x}{\partial \beta}\right) = \text{Sign}\{n_i g(n_z + \beta - n_z \beta)^2 - b(n_i + 1)(n_z + 1)^2\} \]

For \( \beta \geq 0.5 \), we get

\[ b(n_i + 1)(n_z + 1)^2 > n_i g(1 - \beta + n_z \beta)(n_z + \beta - n_z \beta) \]

\[ \geq n_i g(n_z + \beta - n_z \beta)^2 \]

from (12). Thus, we obtain \( \frac{\partial x}{\partial \beta} < 0 \) from (16) and (17) for \( \beta \geq 0.5 \).

For \( \beta < 0.5 \), \( \frac{\partial x}{\partial \beta} < 0 \) holds only if

\[ b(n_i + 1)(n_z + 1)^2 > n_i g(n_z + \beta - n_z \beta)^2 \]

which ensures that the condition in (12) satisfies. (18) can be rewritten as

\[ \frac{b(n_i + 1)}{n_i} > \frac{g(n_z + \beta - n_z \beta)^2}{(n_z + 1)^2} \]

It can be seen that the left-hand side of (19) decreases with \( n_i \) while the right-hand side decreases with \( \beta \) and increases with \( g \) and \( n_z \) if \( \beta < 0.5 \). Hence, if \( \beta < 0.5 \), the greater \( \beta \) and the smaller \( n_i \), \( n_z \), and \( g \) are, the more likely that \( x \) decreases with \( \beta \).

**Proposition 3** If an increase in the spillover occurs,
the quantity of each tier and each firm and the profit of each manufacturer increases if $\beta < 0.5$ and decreases if $\beta > 0.5$.

**Proof** It is easily to get
\[
\text{Sign}\left\{\frac{\partial \Pi}{\partial \beta} \right\} = \text{Sign}\left\{\frac{\partial q_1}{\partial \beta} \right\} = \text{Sign}\left\{\frac{\partial q_2}{\partial \beta} \right\} = \text{Sign}\left\{1 - 2\beta \right\}
\]
which leads to Proposition 3 immediately.

**Proposition 4** (a) Each supplier’s profit decreases with the spillover if $\beta \geq \frac{2n_2 - 1}{3(n_2 - 1)}$. (b) It can not generally concluded how the profit varies with the spillover if $\beta < \frac{2n_2 - 1}{3(n_2 - 1)}$. However, the greater $g$, $n_1$, $n_2$ and $|0.5 - \beta|$ are, the more likely that the profit decreases with $\beta$.

**Proof** Partially differentiating (13) with respect to $\beta$ leads to
\[
\text{Sign}\left\{\frac{\partial \Pi}{\partial \beta} \right\} = \text{Sign}\left\{\frac{\partial q_1}{\partial \beta} \right\} = \text{Sign}\left\{\frac{\partial q_2}{\partial \beta} \right\} = \text{Sign}\left\{1 - 2\beta \right\}
\]
and the condition in (12), we get
\[
\frac{\partial q_2}{\partial n_2} < 0.
\]

**Proposition 5** The quantity of each tier and each supplier increases with $n_1$. Each incumbent supplier’s quantity decreases if new suppliers enter the market, i.e., $n_2$ increases.

**Proof** It is easy to get
\[
\frac{\partial Q}{\partial n_1} > 0, \quad \frac{\partial q_1}{\partial n_1} > 0, \quad \frac{\partial q_2}{\partial n_2} < 0
\]
and the condition in (12), we get
\[
\frac{\partial q_2}{\partial n_2} < 0.
\]

**Proposition 6** (a) The total quantity of each tier increases with $n_2$ if $\beta = 0.5$. (b) It can not generally concluded how the number of the suppliers affects the quantity of each tier if $\beta \neq 0.5$. However, the greater $n_1$, $n_2$ and $|0.5 - \beta|$ are, the more likely that the total quantity decreases with $n_2$.

**Proof** It is easy to get
\[
\text{Sign}\left\{\frac{\partial Q}{\partial n_2} \right\} = \text{Sign}\left\{\xi_2 \right\},
\]
\[
\xi_2 = b(n_1 + 1)(n_2 + 1)^2 - n_1 g[n_2^2 - (3n_2 + 1)(n_2 - 1)(1 - \beta)\beta]
\]
If $\beta = 0.5$, we can get $\xi_2 > 0$ from condition (12) and hence
\[
\frac{\partial Q}{\partial n_2} > 0.
\]
If $\beta \neq 0.5$, noting that
\[
\text{Sign}\left\{\frac{\partial q_2}{\partial \beta} \right\} = \text{Sign}\left\{1 - 2\beta \right\}
\]
Therefore, the greater $g$, $n_1$, $n_2$ and $|0.5 - \beta|$ are, the more likely that the profit decreases with $\beta$. 
\[ n_1 g[n_2^2 - (3n_2 + 1)(n_2 - 1)(1 - \beta)\beta] > n_1 g(1 - \beta + n_2\beta)(n_2 + \beta - n_2\beta) \]

we can not unambiguously sign \( \xi_1 \) from condition (12).

However, if

\[ \varphi_2 = b - \frac{n_1 g n_2^2 - (3n_2 + 1)(n_2 - 1)(1 - \beta)\beta}{n_1 + 1} > 0 \]

we can get \( \frac{\partial Q}{\partial n_2} > 0 \). Noting that \( \frac{\partial \varphi_2}{\partial n_1} < 0 \), \( \frac{\partial \varphi_2}{\partial n_2} < 0 \) and \( \text{Sign}\{\frac{\partial \varphi_2}{\partial n_2}\} = \text{Sign}\{1 - 2\beta\} \), we know that the greater \( n_1 \), \( n_2 \) and \( |0.5 - \beta| \) are, the more likely that \( Q \) decreases with \( n_2 \).

**Proposition 7** It can not generally concluded how the number of the manufacturers affects each incumbent manufacturer’s quantity. However, the greater \( n_2 \) and \( |0.5 - \beta| \) and the smaller \( g \) are, the more likely that each incumbent manufacturer’s quantity decreases with \( n_1 \).

**Proof** It is easy to get

\[ \text{Sign}\{\frac{\partial q_1}{\partial n_1}\} = \text{Sign}\{\xi_2\} \]

\[ \xi_2 = g(1 - \beta + n_2\beta)(n_2 + \beta - n_2\beta) - b(n_2 + 1)^2 \]  (21)

We can not unambiguously indicate the sign of \( \xi_2 \) from (12) and (21). However, if

\[ g(1 - \beta + n_2\beta)(n_2 + \beta - n_2\beta) - b(n_2 + 1)^2 < 0 \]

i.e. \( \varphi_3 = \frac{g(1 - \beta + n_2\beta)(n_2 + \beta - n_2\beta)}{(n_2 + 1)^2} - b < 0 \), we have \( \frac{\partial \varphi_3}{\partial n_1} < 0 \). Noting that \( \frac{\partial \varphi_3}{\partial g} > 0 \), \( \frac{\partial \varphi_3}{\partial n_2} < 0 \) and

\[ \text{Sign}\{\frac{\partial \varphi_3}{\partial n_2}\} = \text{Sign}\{1 - 2\beta\} \], we know that the greater \( n_2 \) and \( |0.5 - \beta| \) and the smaller \( g \) are, the more likely that \( q_1 \) decreases with \( n_1 \).

### 4. R&D Cooperation

In this section, we consider the case that the suppliers cooperate in R&D and remain competition in production. Each supplier \( j \) chooses its R&D expenditure \( x_j \) to maximize \( \Pi_{z,j} + \lambda \sum_{m \neq j} \Pi_{z,m} \), where \( \lambda \in [0, 1] \) captures all possible degrees of R&D cooperation. In the extreme case of full cooperation (when \( \lambda = 1 \)), each supplier chooses its R&D expenditure level to maximize their joint profits.

The decisions of the manufactures and suppliers on quantities keep the same as R&D competition in Section 2. However, supplier \( j \) chooses \( x_j \) to maximize

\[ \Pi_{z,j} + \lambda \sum_{m \neq j} \Pi_{z,m} \]. From (10), we get

\[
\frac{\partial \Pi_{z,j}}{\partial x_j} = \frac{n_1}{b(n_1 + 1)(n_2 + 1)} \left[ (a - v_i - v_i^0) + (n_2 + \beta - n_2\beta)\sqrt{g_{x_i}} + (2\beta - 1)\sum_{m=1}^{n_2} \sqrt{g_{x_m}} \right] \left(2\beta - 1\right)g_{x_j} \sqrt{g_{x_j}}
\]

(22)

\[
\frac{\partial \sum_{m=1}^{n_2} \Pi_{z,m}}{\partial x_j} = \frac{n_1}{b(n_1 + 1)(n_2 + 1)} \left[ (n_2 - 1)(a - v_i - v_i^0) + (n_2 + \beta - n_2\beta)\sqrt{g_{x_j}} + (2\beta - 1)\sum_{m=1}^{n_2} \sqrt{g_{x_m}} \right] \left(2\beta - 1\right)g_{x_j} \sqrt{g_{x_j}}
\]

(23)

Supplier \( j \) will choose its R&D investment level according to the following first-order condition

\[
\frac{\partial (\Pi_{z,j} + \lambda \sum_{m \neq j} \Pi_{z,m})}{\partial x_j} = 0
\]
We can get the first-order condition as the function of each supplier’s R&D level from (22) and (23). Since the suppliers are identical, the subscript of \( x \) can be deleted and the first-order condition is reduced to

\[
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\[
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\]
\[
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\]
\[
βλββ
\]
\[
βλ
\]
\[
β
\]

Then we get

\[
01 = \frac{n_g[(n_1 + \beta - n_2\beta) + \lambda(n_2 - 1)(2\beta - 1)]}{(a - v_1 - v_2) + (1 - \beta + n_2\beta)(2\beta - 1)} - 1 = 0
\]

We need the following condition

\[
n_g[(n_1 + \beta - n_2\beta) + \lambda(n_2 - 1)(2\beta - 1)]
\]

\[
(n_2 + \beta - n_2\beta) + \lambda(n_2 - 1)(2\beta - 1)
\]

> 0

\[
\frac{n_g[(n_1 + \beta - n_2\beta) + \lambda(n_2 - 1)(2\beta - 1)]}{(a - v_1 - v_2) + (1 - \beta + n_2\beta)(2\beta - 1)}
\]

We can also get

\[
Q_i^* = Q^n_i = Q^n = \frac{n_1n_2(n_1 + 1)(a - v_1 - v_2)}{b(n_1 + 1)(n_2 + 1)^2 - n_g(1 - \beta + n_2\beta)[(n_2 + \beta - n_2\beta) + \lambda(n_2 - 1)(2\beta - 1)]}
\]

We obtain

\[
Q_i^* = Q^n_i = Q^n = \frac{n(n_1 + 1)(a - v_1 - v_2)}{b(n_1 + 1)(n_2 + 1)^2 - n_g(1 - \beta + n_2\beta)[(n_2 + \beta - n_2\beta) + \lambda(n_2 - 1)(2\beta - 1)]}
\]

**Proposition 8** The R&D expenditures increases with \( \lambda \), the degree of R&D cooperation.

**Proof** The proof is immediately followed from the fact that

\[
\frac{\partial \sqrt{gx^*}}{\partial \lambda} > 0
\]

Noting that \( \lambda = 0 \) is actually the case that the suppliers do not cooperate in R&D, we know from Proposition 8 that the R&D investment level under cooperation is higher than that under competition. Furthermore, the larger the degree of cooperation is, the larger the gap of R&D level between cooperation and competition is. The gap reaches the largest value if the suppliers engage in full cooperation.

**Proposition 9** (a) The R&D investments increase with the spillover if the degree of cooperation is appropriately large, i.e., \( \lambda \geq 0.5 \). (b) We can not generally indicate how the change in the spillover affects the R&D investment if \( 0 < \lambda < 0.5 \). However, the greater \( \lambda \) and \( g \) and the smaller \( n_1 \) and \( \beta \) are, the more likely that the R&D investments increase with \( \beta \). If \( \beta < 0.5 \), the greater \( n_2 \) is, the more likely that the R&D investment increases with \( \beta \). If \( \beta > 0.5 \), the
\[
\frac{\partial \phi_1}{\partial g} > 0, \quad \frac{\partial \phi_1}{\partial n_1} < 0
\]

\[\text{Sign}\left\{ \frac{\partial \phi_1}{\partial \beta}\right\} = \text{Sign}\{-(n_1 - 1)(2\lambda - 1)^2\}\]

\[\text{Sign}\left\{ \frac{\partial \phi_1}{\partial \lambda}\right\} = \text{Sign}\{(n_2 + \beta - n_2\beta) + (n_1 - 1)(2\beta - 1)(1 - \lambda)\}\]

\[\text{Sign}\left\{ \frac{\partial \phi_1}{\partial \alpha}\right\} = \text{Sign}\{(1 - 2\beta)(2\lambda - 1)^2\}\]

Since \(0 < \lambda < 0.5\), we can readily get \(\frac{\partial \phi_1}{\partial \beta} < 0\) and \(\frac{\partial \phi_1}{\partial \lambda} > 0\). In addition, \(\frac{\partial \phi_1}{\partial n_2} > 0\) if \(\beta < 0.5\) and \(\frac{\partial \phi_1}{\partial n_2} < 0\) if \(\beta > 0.5\).

**Proposition 10** When the degree of R&D cooperation changes, the quantity of each tier and each firm increases if \(\beta > 0.5\), decreases if \(\beta < 0.5\) and remains unchanged if \(\beta = 0.5\).

**Proof** From (25), (26) and (27), we have

\[\text{Sign}\left\{ \frac{\partial Q^*}{\partial \lambda}\right\} = \text{Sign}\left\{ \frac{\partial q^*_1}{\partial \lambda}\right\} = \text{Sign}\{2\beta - 1\}\]

which leads to the proposition immediately.

**Proposition 11** (a) If \(\lambda \geq \frac{n_2 - 1}{3n_2 - 1}\), the quantity of each tier and each firm increases with \(\beta\). (b) If \(\lambda < \frac{n_2 - 1}{3n_2 - 1}\), the quantity may increase or decrease with \(\beta\). The smaller \(\beta\) is, the more likely that the quantity increases with \(\beta\).

**Proof** From (25), (26) and (27), we get

\[\text{Sign}\left\{ \frac{\partial Q^*}{\partial \beta}\right\} = \text{Sign}\left\{ \frac{\partial q^*_1}{\partial \beta}\right\} = \text{Sign}\{\xi_4\}\]

\[\xi_4 = (n_2 - 1) - \lambda(n_2 - 3) + 2(n_2 - 1)(2\lambda - 1)\beta\]

(a) If \(\lambda \geq 0.5\), we have

\[\xi_4 \geq (n_2 - 1) - \lambda(n_2 - 3) > 0\]

If \(\lambda < 0.5\), we can obtain

\[\xi_4 \geq (n_2 - 1) - \lambda(n_2 - 3) + 2(n_2 - 1)(2\lambda - 1)\]

\[= (3n_2 - 1)\lambda - (n_2 - 1)\]

(30)

If \(\frac{n_2 - 1}{3n_2 - 1} \leq \lambda < 0.5\), from (30) we get \(\xi_4 \geq 0\) and the equation holds only when \(\beta = 1\).

(b) If \(\lambda < \frac{n_2 - 1}{3n_2 - 1}\), we obtain

\[\frac{\partial \xi_4}{\partial \beta} = 2(2n_2 - 1)(2\lambda - 1) < 0\]

This completes the proof.

5. Conclusion

This paper has derived a model with upstream R&D in a two-tier supply chain. We considered R&D cooperation as well as R&D competition. Under R&D competition, we studied how the changes in R&D spillover, R&D efficiency and the numbers of the suppliers and the manufacturers affect R&D investments, quantities and profits. Under R&D cooperation, we showed that how R&D investments and quantities change with R&D spillover and the degree of R&D cooperation. The main results in the paper are given in eleven propositions.

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References


